

Note about leptogenesis from gravity waves in models of inflation

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Abstract

We show how the mechanism recently proposed by Alexander, Peskin, and Sheikh-Jabbari, in which the observed cosmic matter-antimatter asymmetry comes from the contribution of the cosmological tensor perturbations to the gravitational anomaly in the Standard Model, does not work. This holds when taking into account the commutation relations for the creation-annihilation operators associated to the gravitational waves and the geometrical properties of the polarization tensors.

Quite recently Alexander, Peskin, and Sheikh-Jabbari [1] described a mechanism to generate the cosmological matter-antimatter asymmetry from the gravitational anomaly in the Standard Model [2]:

$$\partial_\mu J_\ell^\mu = \frac{1}{16\pi^2} R\tilde{R}, \quad (1)$$

where a non-zero value for the gravitational term

$$R\tilde{R} = \epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\rho\sigma} R_{\gamma\delta}{}^{\rho\sigma}, \quad (2)$$

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could lead to a non-conserved leptonic current

$$J_\ell^\mu = \bar{\ell}_i \gamma^\mu \ell_i + \bar{\nu}_i \gamma^\mu \nu_i. \quad (3)$$

It was claimed in [1] that a contribution to $R\tilde{R}$ of definite sign might be generated by the tensor cosmological perturbations, produced during inflation, if the inflaton field contained a CP-odd component. This could be possible if the inflaton were a complex modulus field and the imaginary part ϕ of this field could couple to gravity through an interaction of the type

$$\Delta\mathcal{L} = F(\phi)R\tilde{R}, \quad (4)$$

being F odd in ϕ . This kind of interaction could be the source of the observable parity-violation in the cosmic microwave background [3] or might cause an asymmetric evolution of the two polarization states of the tensor-type perturbation as it is shown in [4]. A simple form for $F(\phi)$ with the correct scaling is [1]

$$F(\phi) = \frac{1}{16\pi^2 M_{Pl}} \mathcal{N}\phi, \quad (5)$$

where the M_{Pl} in the denominator is approximately the string scale.

To quantitatively estimate the lepton number produced during inflation [5], it is necessary to compute the production of gravitational waves under the influence of the coupling in Eq. (4). When the inflaton field has a slowly-rolling nonzero classical value, the coupling in the Eq. (4) can lead to quantum fluctuations of the gravitational field that, treated to second order, generate a nonzero right-hand side for the expression in Eq. (1).

The general form of the cosmological tensor perturbations in a Friedman-Robertson-Walker Universe can be parameterized as

$$ds^2 = dt^2 - a^2(t) (\delta_{ij} + h_{ij}) dx^i dx^j, \quad (6)$$

where h_{ij} denotes the tensor fluctuations of the metric. Thus, the contribution of the tensor perturbations to $R\tilde{R}$, up to second order in h_{ij} , is

$$\begin{aligned} R\tilde{R} = & - \frac{2}{a^3} \epsilon^{ijk} \left[\left(\frac{\partial^2}{\partial_t \partial t} h_{jm} \frac{\partial^2}{\partial_m \partial_i} h_{kl} - \frac{\partial^2}{\partial_t \partial t} h_{jm} \frac{\partial^2}{\partial_t \partial_i} h_{km} \right) \right. \\ & \left. + a^2 \left(\frac{\partial^2}{\partial t^2} h_{jl} \frac{\partial^2}{\partial_i \partial t} h_{lk} \right) + \frac{1}{2} \frac{\partial}{\partial t} a^2 \left(\frac{\partial}{\partial t} h_{jl} \frac{\partial^2}{\partial_i \partial t} h_{lk} \right) \right], \end{aligned} \quad (7)$$

where we have considered gravity waves moving in an arbitrary direction in the three-space.

By adding the expression in Eq. (4) to the Einstein action and varying with respect to the metric fluctuations, it is possible to find the equations of motion

$$M_{Pl}^2 \square h_{ij} = -\frac{4}{a} \epsilon^{ilk} \frac{\partial^2}{\partial_k \partial t} h_{jl} \left(F'' \dot{\phi}^2 + 2HF' \dot{\phi} \right), \quad (8)$$

where we have dropped terms with third-order derivatives of h_{ij} and neglected the acceleration of the inflaton field $\ddot{\phi}$. As it was described in [3] the coupling in Eq. (4) makes the term $R\tilde{R}$ different to zero by generating a “cosmological birefringence” during inflation. This cosmological birefringence assigns different dispersion relations for the right- and left-handed polarized waves so that the expression in Eq. (7) does not vanish.

Writing the tensor perturbations h_{ij} in conformal time, and expanding them in Fourier modes

$$h_{ij}(\mathbf{x}, \tau) = \frac{\sqrt{2}}{M_{Pl}} \int \frac{d^3k}{(2\pi)^{3/2}(2\omega_k)^{1/2}} \sum_p \left[e^{i\mathbf{k} \cdot \mathbf{x}} h(p, \mathbf{k}, \tau) \epsilon_{ij}(p, \mathbf{k}) a(p, \mathbf{k}) + h.c. \right], \quad (9)$$

where the summation on p is over the two polarization states of the gravitational waves $(+, \times)$, we can calculate the vacuum expectation value of the gravitational operator $R\tilde{R}$ (Eq. (7)) written this time in conformal time:

$$R\tilde{R} = -\frac{2}{a^4} \epsilon^{ijk} \left(\frac{\partial^2}{\partial_l \partial \tau} h_{jm} \frac{\partial^2}{\partial_m \partial_i} h_{kl} - \frac{\partial^2}{\partial_l \partial \tau} h_{jm} \frac{\partial^2}{\partial_l \partial_i} h_{km} + \frac{\partial^2}{\partial \tau^2} h_{jl} \frac{\partial^2}{\partial_i \partial \tau} h_{lk} \right). \quad (10)$$

Note that in Eq. (9) there are two creation (annihilation) operators, one for each polarization state, with the usual commutation relations:

$$\begin{aligned} [a(p, \mathbf{k}), a^\dagger(p', \mathbf{k}')] &= \delta_{pp'} \delta^3(\mathbf{k} - \mathbf{k}'), \\ [a(p, \mathbf{k}), a(p', \mathbf{k}')] &= 0, \\ [a^\dagger(p, \mathbf{k}), a^\dagger(p', \mathbf{k}')] &= 0. \end{aligned} \quad (11)$$

The presence of the creation-annihilation operators and their commutation relations was not considered by Alexander *et. al.* [1], and that led them to conclusions very different from ours about the vacuum expectation value of the operator $R\tilde{R}$ as we will show.

Note also that the polarization tensors behave similarly to the vector polarizations for the electromagnetic field: the polarization tensor $\epsilon_{ij}(p, \mathbf{k})$ is “orthogonal” to the propagation direction \mathbf{k} of the gravitational wave:

$$k_i \epsilon_{ij}(p, \mathbf{k}) = 0. \quad (12)$$

In addition, the polarization tensors are “orthonormal”:

$$\epsilon_{ij}^*(p, \mathbf{k}) \epsilon_{ij}(p', \mathbf{k}) = 2\delta_{pp'}. \quad (13)$$

This, of course, is equivalent to say that the “cross” product between two different polarization tensors results in a unitary vector in the direction of propagation of the gravitational waves:

$$\begin{aligned}\epsilon^{ilk}\epsilon_{ij}^*(+, \mathbf{k})\epsilon_{jl}(\times, \mathbf{k}) &= -\epsilon^{ilk}\epsilon_{ij}^*(\times, \mathbf{k})\epsilon_{jl}(+, \mathbf{k}) = 2\frac{k_k}{|\mathbf{k}|}, \\ \epsilon^{ilk}\epsilon_{ij}^*(+, \mathbf{k})\epsilon_{jl}(+, \mathbf{k}) &= \epsilon^{ilk}\epsilon_{ij}^*(\times, \mathbf{k})\epsilon_{jl}(\times, \mathbf{k}) = 0.\end{aligned}\tag{14}$$

We have proved the expressions in Eq. (14) by using properties of the rotational transformations.

Thus, the vacuum expectation value $\langle 0 | R\tilde{R} | 0 \rangle$ can now be calculated:

$$\langle 0 | R\tilde{R} | 0 \rangle = -\frac{2}{a^4} \epsilon^{ijk} \left\langle 0 \left| \frac{\partial^2}{\partial_l \partial \tau} h_{jm} \frac{\partial^2}{\partial_m \partial_i} h_{kl} - \frac{\partial^2}{\partial_l \partial \tau} h_{jm} \frac{\partial^2}{\partial_l \partial_i} h_{km} + \frac{\partial^2}{\partial \tau^2} h_{jl} \frac{\partial^2}{\partial_i \partial \tau} h_{lk} \right| 0 \right\rangle.\tag{15}$$

Using the properties of the polarization tensors, and the commutation relations of the creation-annihilation operators it is easy to see how each of the three contribution in Eq. (15) vanishes. The contributions from the second and third term vanish as the bracket $\langle 0 | \dots | 0 \rangle$ implies the same polarization state for the creation-annihilation operators involved. This in turn means that we are left with the products $\epsilon^{ijk}\epsilon_{jm}^*(p, \mathbf{k})\epsilon_{km}(p, \mathbf{k})$ which, according to the Eq. (14), are zero. Finally, the contribution from the first term vanishes as the spacial derivatives involved lead to products of the form $k_i\epsilon_{ij}(p, \mathbf{k})$ which are, of course, zero (see Eq. (12)).

As a conclusion we can say that, contrary to what was claimed by Alexander *et al.* [1] who found a quite positive result, the matter-antimatter asymmetry cannot be generated from the contribution of the tensor cosmological perturbations to the gravitational anomaly in the Standard Model.

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